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Influence of Entire Turbulent Spectrum on Modeling of Breakup in Liquid-Liquid Systems

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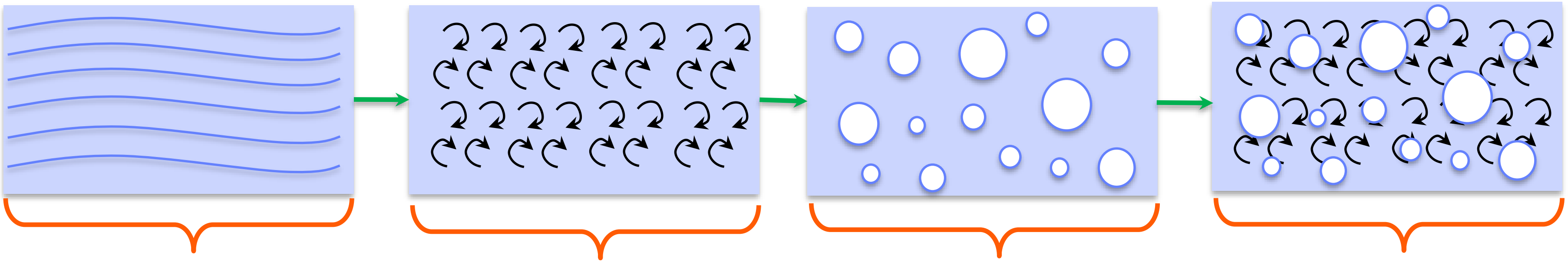
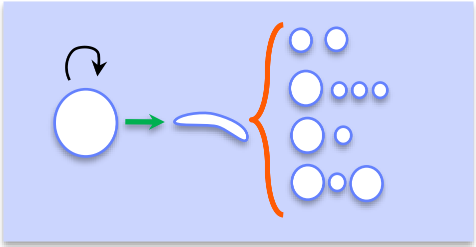
Problem Definition



Laminar Flow | Turbulent Flow | Multiphase Flow | Multiphase-Turbulent Flow

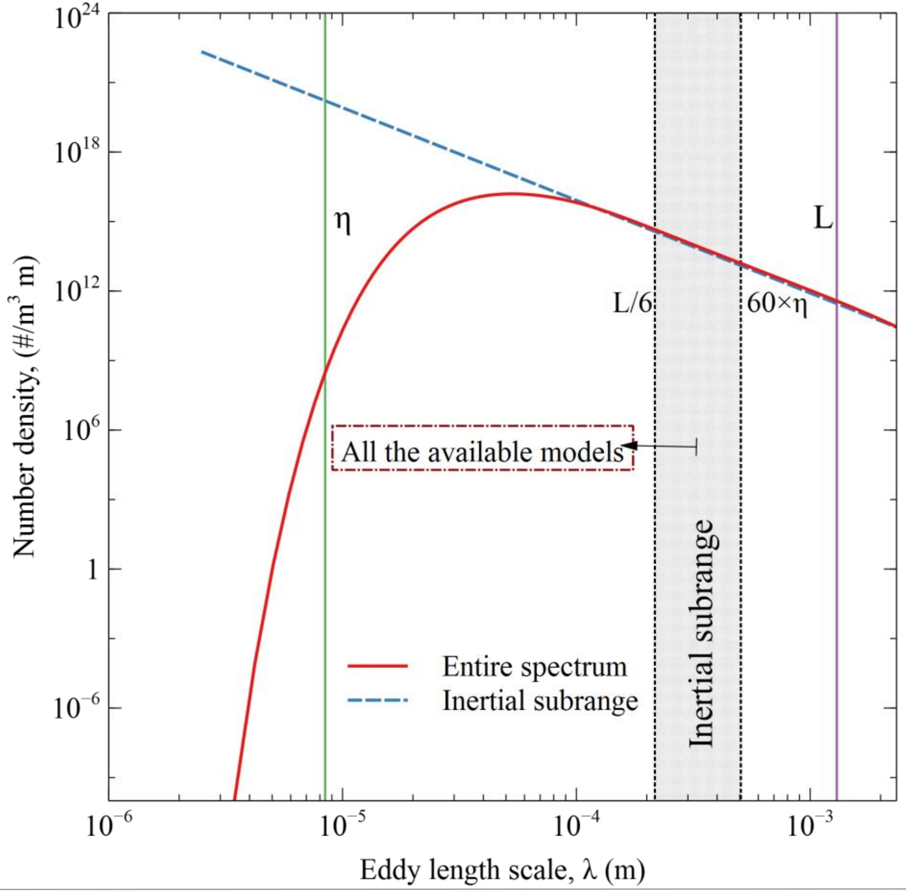
$$\frac{\partial n(v,t)}{\partial t} + \nabla \cdot (n(v,t)\overline{\boldsymbol{U}}) = B_{breakup} + D_{breakup} + B_{coalescence} + D_{coalescence}$$



✓ **A Numerical Modeling Strategy for This Physical Phenomenon.**

Turbulent Spectrum

- Different regions of the spectrum,
- Available models: breakup only occurs in the inertial subrange,
- **Lack of modeling strategies for other subranges!**



Formulation of a Breakup Rate Model

$$\Omega_s = \underbrace{Frequency\ of\ interaction}_{\dot{\omega}(d_0,\lambda)} \times \underbrace{Efficiency\ of\ interaction}_{P(d_0,\lambda)}$$

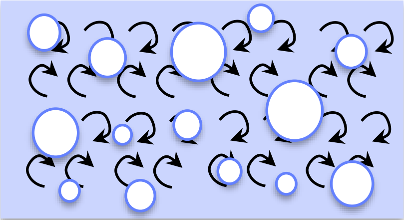
$$\dot{\omega}(d_0,\lambda) = \underbrace{number\ density\ of\ eddies}_{n_\lambda} \times \underbrace{volume\ of\ droplet}_{V_b} \times \underbrace{life\ time\ of\ eddies}_{\tau_\lambda}$$

$$P(d_0,\lambda) = -\max\left[\chi_{\substack{interfacial\\energy}}, \chi_{\substack{disruptive\\stress}}\right]$$

$\chi_{\substack{interfacial\\energy}}$: available energy exceeds the interfacial energy

$\chi_{\substack{disruptive\\stress}}$: disruptive stresses exceed cohesive stresses

$$\boldsymbol{\Omega(d_0) = \int_{\lambda_{min}}^{\lambda_{max}} \omega(d_0,\lambda)P(d_0,\lambda)d\lambda}$$



Entire spectrum vs. Inertial subrange

- **Approximating the eddy velocity using the second-order structure function:**

  - **Kolmogorov for inertial subrange:**

$$\langle[\delta v]^2\rangle(\lambda) = C \times (\varepsilon\lambda)^{2/3}$$

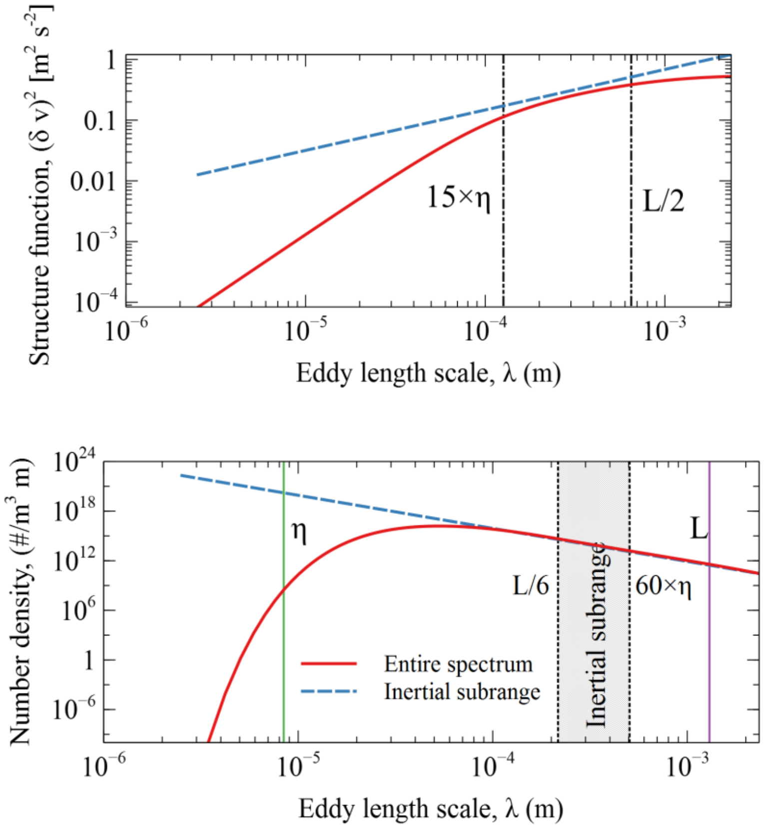
  - **Davidson for entire spectrum:**

$$\langle[\delta v]^2\rangle(\lambda) = \frac{4}{3}\int_{-\infty}^{\infty} E(\kappa)\left[1 - 3\left\{\frac{\sin(\kappa\lambda)}{(\kappa\lambda)^3} + \frac{\cos(\kappa\lambda)}{(\kappa\lambda)^2}\right\}\right] d\kappa$$

Entire spectrum vs. Inertial subrange (cont.)

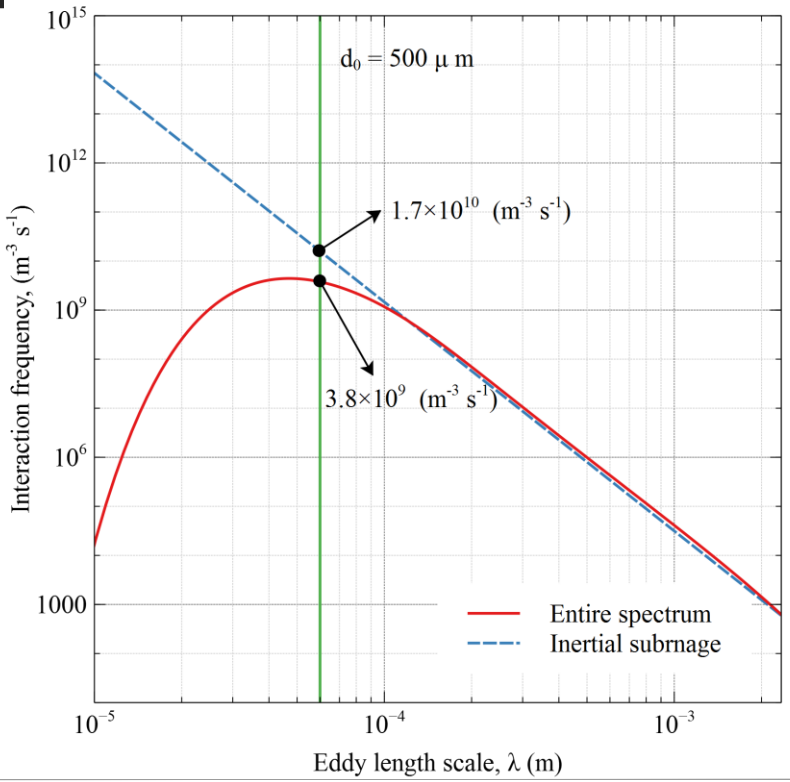
- **Practical definition of inertial subrange based on the overlapping zone,**

- **Improved understanding of the contribution of turbulent energy spectrum.**



Upgrading the breakup model

- Turbulent eddy velocity,
- Number density of eddies,
- Life time of eddies,
- Comparing interaction frequency using two formulations



Upgrading the breakup model (cont.)

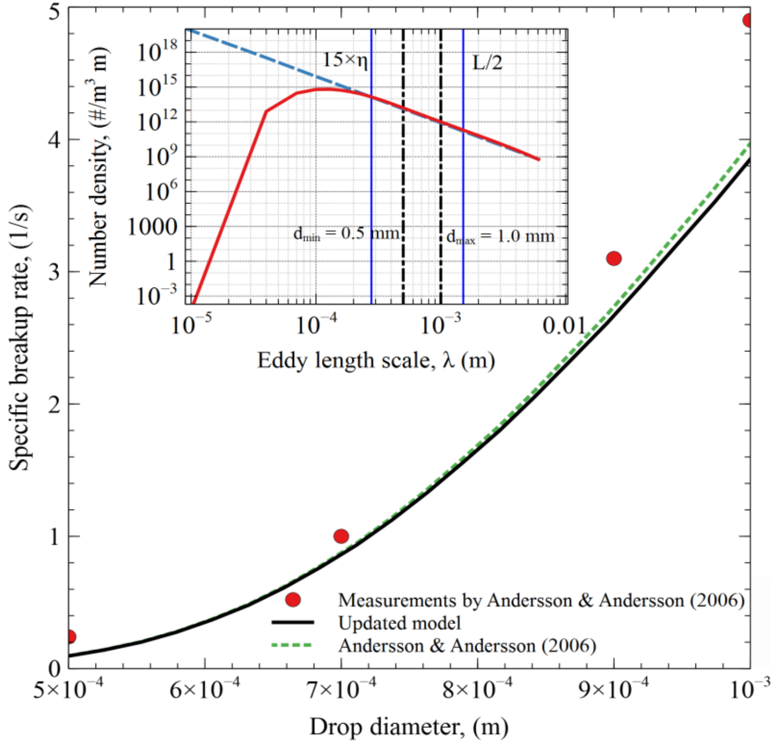
- **Updated model should work for both the inertial subrange and outside this region.**

- **Example 1. Inertial subrange (Direct measurements)**

Phases (continuous-dispersed)	Droplet diameter [m]	Exp. Breakup rate [m^{-3} s^{-1}]	Operational conditions			Turbulent details	
			$\sigma\ [N\ m^{-1}]$	$\rho_d\ [kg\ m^{-3}]$	$\mu_d [Pa\ s]$	$\varepsilon\ [m^2 s^{-3}]$	8.5
Water-Dodecane (Andersson and Andersson, 2006)	5×10^{-4}	0.24				$k\ [m^2 s^2]$	0.087
	7×10^{-4}	1.0				$L\ [\ m\ ]$	3.02×10^{-3}
	9×10^{-4}	3.1	0.053	750.0	0.0015	$\eta\ [m]$	1.86×10^{-5}
	1×10^{-3}	4.9				$\lambda_T\ [m]$	3.21×10^{-4}
						$Re_L\ [-]$	889.35

Example 1. Inertial subrange

- **Drop diameters within the inertial subrange.**
- **Similar predictions for both models (no surprise!).**
- **<u>The updated model works for the inertial subrange.</u>**



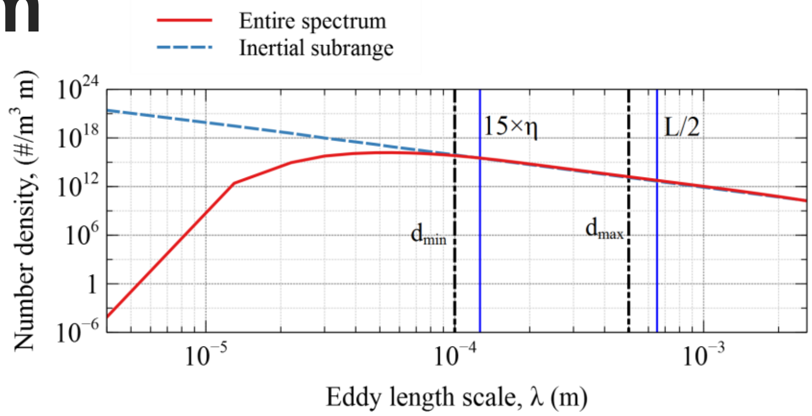
Example 2. Entire spectrum

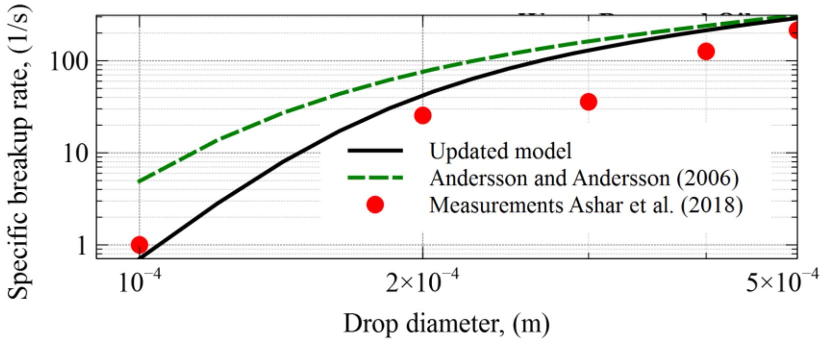
- **Direct measurements of breakup rates:**

Phases (continuous-dispersed)	Droplet diameter [m]	Exp. Breakup rate [m^{-3} s^{-1}]	Operational conditions			Turbulent details	
			$\sigma\ [N\ m^{-1}]$	$\rho_d\ [kg\ m^{-3}]$	$\mu_d [Pa\ s]$	$\varepsilon\ [m^2 s^{-3}]$	200.0
Water-Rapeseed oil	1×10^{-4}	0.0	0.020	920.0	0.0699	$k\ [m^2 s^2]$	0.4
	2×10^{-4}	25.47				$L\ [\ m\ ]$	0.0013
	3×10^{-4}	35.78				$\eta\ [m]$	8.43×10^{-6}
	4×10^{-4}	126.32				$\lambda_T\ [m]$	1.42×10^{-4}
	5×10^{-4}	214.29				$Re_L\ [-]$	796.81

Example 2. Entire spectrum (cont.)

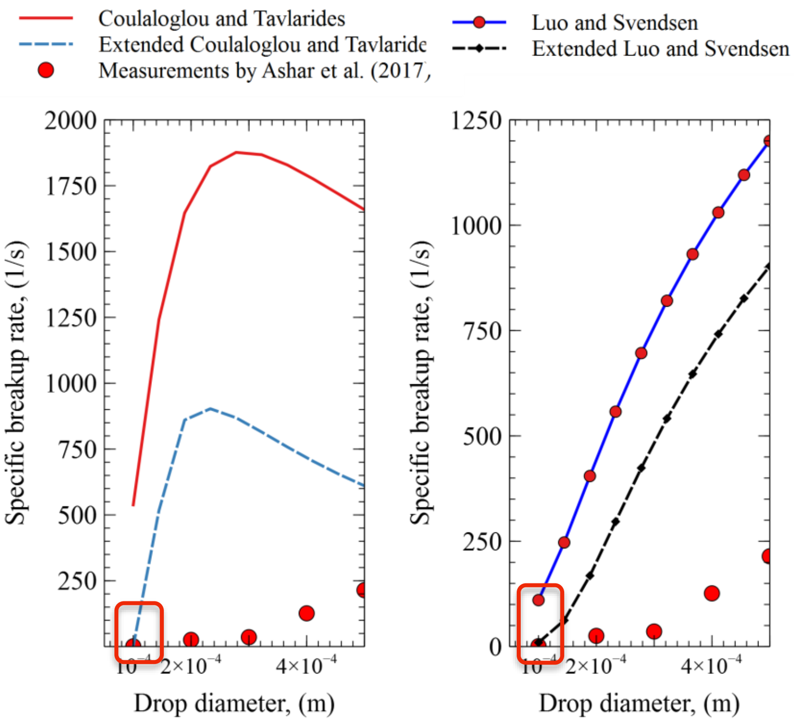
- A unique data point outside the inertial subrange.
- The updated model **starts to show improvements toward the dissipation subrange.**
- The updated model works outside the inertial subrange.





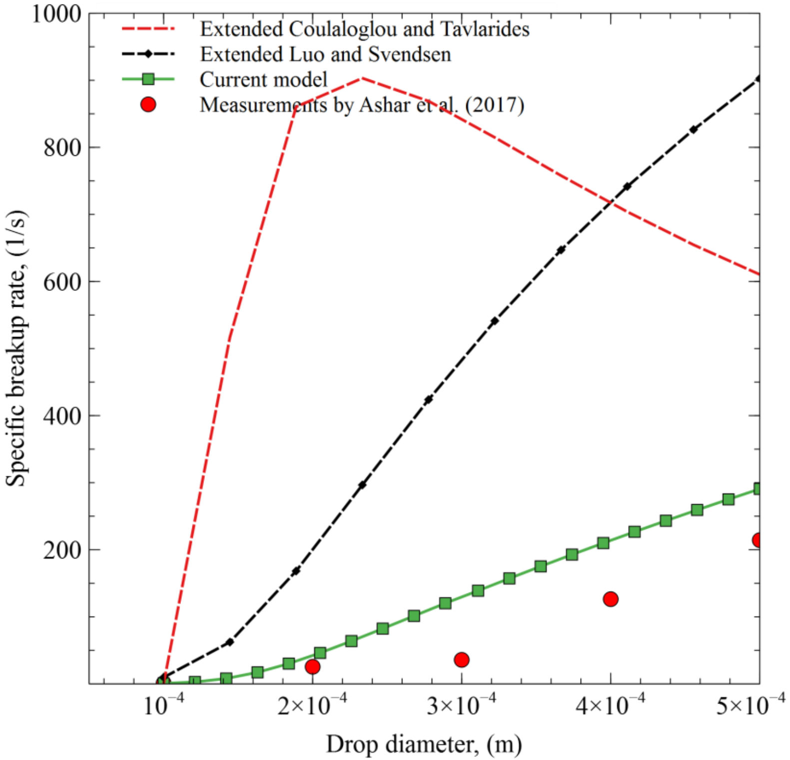
Upgrading other breakup models

- **Commonly used models,**
- **Improvement for the dissipation subrange,**
- **The model structure might not accommodate further improvements.**



Upgrading other breakup models (cont.)

- **Model extension depending on model formulation,**
- **Higher predictive capabilities toward dissipation subrange,**
- **Extension of breakup kernels: when the droplet diameters are not limited to the inertial subrange.**



Concluding Remarks

- **Importance of entire turbulent spectrum for modeling fluid particles breakup.**

- **A more realistic representation of turbulent structures (numbers, velocity, and inertial subrange).**

- **Validated model confirms the benefits of the entire turbulent spectrum for breakup formulation.**